Design for Temperature and Thermal Buckling Constraints Employing a Noneigenvalue Formulation

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Re-entry vehicles such as the Space Shuttle are subject to severe heating loads during significant portions of their flight trajectories. The conventional design of such vehicles is composed of several sequential design tasks. First, the trajectory is optimized to reduce the heat load and aerodynamic loads on the vehicle. Second, the structure is designed to carry the loads. Third, a thermal protection system is designed to absorb the heat loads without exceeding the allowable temperature in the infrastructure. Such a design procedure is very time consuming and also may not always yield the optimum design. In particular, there are situations where it is advantageous to design the thermal protection system to provide more protection so that temperatures in the infrastructure are well below the allowable. While the mass of the thermal protection system is increased, the mass of the structure may be reduced because the stress allowables are higher at lower temperatures. The present paper continues this line of investigation for thermal buckling constraints. The enforcement of thermal buckling constraints is treated in a novel way which does not require any eigenvalue analysis. An example of an application to the design of a wing bay on the Orbiter of the Space Shuttle is presented. It demonstrates the design procedure and some of its benefits.

Nomenclature

D = diagonal matrix in Gauss-Doolittle decomposition, Eq. (9)
d_{ii} = components of D matrix
E_{ij} = constitutive matrix coefficients for orthotropic plate
g_i = constraint function
I_s = set of structural design variables
I_T = set of thermal design variables
K = total stiffness matrix, Eq. (8)
K_g = geometrical stiffness matrix
K_{g0} = geometrical stiffness matrix per unit λ, Eq. (2)
K_0 = initial stiffness matrix
k_{ij} = components of total stiffness matrix
L = lower triangular matrix in Gauss-Doolittle decomposition, Eq. (9)
m = total mass of the system
n = number of design variables
T = temperature
v = vector of design variables
v_0 = nominal vector of design variables
v_i = components of v
v_{0i} = components of v_0
x = buckling mode
λ = buckling eigenvalue
σ = stress
ρ = density
$(\ )'$ = differentiation with respect to a design variable

Introduction

THE recent interest in the application of optimization procedures to structural design problems is evidenced by the hundreds of references contained in recent surveys of the field (e.g., Ref. 1). One result of this intensive focus on structural optimization is a realization that for a truly optimum system the designer often should not treat the design of the structure separately from the design of the entire system. The interaction of the structure and the aerodynamics of the vehicle recently has been of particular interest in aerospace applications. Thus it has been shown that through the use of aeroelastic tailoring with composite materials the structure may be designed to improve aerodynamic performance (e.g., Ref. 2). Another important area of interaction which has not received as much attention, and which is the subject of the present paper, is the interaction of thermal and structural considerations in the design of insulated structures.

Re-entry vehicles such as the Space Shuttle are subject to severe heating loads during significant portions of their flight trajectories. The conventional design of such vehicles is composed of several sequential design tasks. First, the trajectory is optimized to reduce the heat load on the vehicle. Second, the structure is designed to carry the loads. Third, a thermal protection system is designed to absorb the heat loads without exceeding the allowable temperature in the infrastructure. Optimization procedures are available to carry out each one of the tasks individually. This includes procedures for trajectory optimization (e.g., Ref. 3), procedures for optimizing insulation (e.g., Refs. 4 and 5), and as noted above, procedures for structural optimization. There have been studies involving combined structural/thermal optimization,[6-10] but it has only been recently demonstrated that the sequential design approach may be inappropriate for re-entry vehicles.[11] There has also been significant interest in the past few years in integrating the processes of structural and thermal design of aerospace vehicles.[12-15]

The structural/thermal interaction that is the subject of Ref. 11 is the deterioration of structural material properties at elevated temperatures, in particular the stress allowables. As a result of this material degradation it is sometimes preferable to use more insulation than is needed to keep the structure from exceeding its maximum allowable temperature. The extra weight of the insulation is recovered with a gain because the weight of the structure may be reduced when it operates at lower temperatures. Additionally, the simultaneous approach to thermal/structural design problems is an important short cut in the overall design process. It also permits a rapid evaluation of the effect of parameters such as allowable temperatures on the overall weight of the system.

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This paper is directed toward another mechanism of interaction between the thermal and structural design—thermal buckling. Some structural panels of the Shuttle Orbiter wing are primarily designed by the constraint of avoiding buckling generated by thermal gradients. To alleviate a thermal buckling problem we may increase the stiffness of the panels or the thickness of the insulation and thus reduce the thermal gradients. A completely integrated thermal/structural design of part of the structure on a re-entry vehicle may become prohibitively expensive because it requires repeated transient thermal analyses. This problem is avoided in this work by adopting the approach of Ref. 15 whereby the dependence of the temperature in the structure is approximated by a linear function of the structural thicknesses and the reciprocals of the insulation thicknesses. The present work also presents a novel approach of treating the thermal buckling constraint. The traditional approach is to treat the buckling problem as an eigenvalue problem and to apply constraints to the eigenvalues. The present approach is to pose the buckling problem as a requirement that the stability matrix be positive definite, and to constrain the diagonal terms on the Gauss-Doolittle factor of the matrix to be positive.

The design approach is demonstrated by a design study of the structure and insulation of a Shuttle Orbiter wing bay.

Buckling Analysis and Sensitivity

The finite element analysis of structural buckling is usually formulated in the following form

$$[K_0 + K_g]x = 0 \tag{1}$$

where K_0 is the stiffness matrix of the structure, K_g the geometric stiffness matrix, and x a buckling mode. The geometric stiffness matrix depends not only on the structure but also on the stresses, and it is customary to write it as

$$K_g = K_g(\sigma) = \lambda K_{g0} \tag{2}$$

where σ is a vector of stresses and λ is some arbitrary scalar parameter. Equation (1) is then written as

$$[K_0 + \lambda K_{g0}]x = 0 \tag{3}$$

which is the classical form of a generalized matrix eigenvalue problem. In many cases it is possible to attach physical significance to λ, particularly if the stresses in the structure can be simply scaled by scaling up the applied loads. In the case of combined mechanical and thermal loads this may not be always possible, so here λ is viewed merely as a mathematical device. If we arbitrarily define $\lambda = 1$ as the value corresponding to the actual loads then the constraint that the structure will not buckle under these loads is expressed as

$$\lambda_i > 1 \qquad i = 1,2,\ldots \tag{4}$$

for all the eigenvalues λ_i of Eq. (3).

For a structural optimization problem subject to buckling constraints it is useful also to have the derivatives of the constraints with respect to design variables. The derivative of an eigenvalue with respect to a design variable v can be found easily by differentiating Eq. (3) with respect to v

$$\left[\frac{dK_0}{dv} + \frac{d\lambda}{dv}K_{g0} + \lambda\frac{dK_{g0}}{dv}\right]x + [K_0 + \lambda K_{g0}]\frac{dx}{dv} = 0 \tag{5}$$

and premultiplying Eq. (5) by x^T to obtain

$$\frac{d\lambda}{dv} = x^T\left[\frac{dK_0}{dv} + \lambda\frac{dK_{g0}}{dv}\right]x \bigg/ (x^T K_{g0} x) \tag{6}$$

Unfortunately, the derivative of the geometric stiffness with respect to the design variable may not be easy to obtain because K_{g0} is a function of the stresses, and the stresses are a function of the temperature T. That is,

$$\frac{dK_{g0}}{dv} = \frac{\partial K_{g0}}{\partial \sigma}\left[\frac{\partial \sigma}{\partial T}\frac{dT}{dv} + \frac{\partial \sigma}{\partial v}\right] + \frac{\partial K_{g0}}{\partial v} \tag{7}$$

The above analysis and sensitivity formulation of buckling constraints is fairly standard, and is reproduced here for comparison with the method proposed below.

The formulation proposed herein states the buckling constraint as the requirement that the sum of the stiffness and geometric stiffness matrices

$$K = K_0 + K_g \tag{8}$$

is positive definite.

This requirement is imposed by using the Gauss-Doolittle factorization of the matrix K

$$K = LDL^T \tag{9}$$

where L is a lower triangular matrix with all diagonal terms being equal to 1 and D is a diagonal matrix. The positive definiteness of the matrix K is guaranteed if all the terms in D are positive

$$d_{ii} \geq 0 \qquad i = 1,2,\ldots \tag{10}$$

In terms of analysis this formulation has a clear advantage over the traditional formulation above, because the Gauss-Doolittle factorization is much cheaper than the solution of an eigenvalue problem. The calculation of sensitivity, however, makes the situation much less clear.

To calculate the derivative of the matrix D with respect to a design variable, Eq. (9) is differentiated

$$\frac{dK}{dv} = \frac{dL}{dv}DL^T + L\frac{dD}{dv}L^T + LD\frac{dL^T}{dv} \tag{11}$$

For an individual term in the matrix, Eq. (11) may be written as

$$k_{ij} = \sum_{n=1}^{j}(\ell'_{in}d_{nn}\ell_{jn} + \ell_{in}d'_{nn}\ell_{jn} + \ell_{in}d_{nn}\ell'_{jn}) \qquad (i \geq j) \tag{12}$$

where a prime denotes differentiation with respect to a design variable v. Equation (12) is rewritten as

$$k'_{ij} = \ell'_{ij}d_{jj} + \ell_{ij}d'_{jj} + \sum_{n=1}^{j-1}(\ell'_{in}d_{nn}\ell_{jn} + \ell_{in}d'_{nn}\ell_{jn} + \ell_{in}d_{nn}\ell'_{jn}) \tag{13}$$

Equation (13) may now be used as a basis for recursive formulas for the evaluation of the derivatives of L and D

$$d'_{ii} = k'_{ii} - \sum_{n=1}^{i-1}(2\ell'_{in}d_{nn}\ell_{in} + \ell^2_{in}d'_{nn}) \tag{14}$$

and

$$\ell'_{ij} = \left[k'_{ij} - \ell_{ij}d'_{jj} - \sum_{n=1}^{j-1}(\ell'_{in}d_{nn}\ell_{jn} + \ell_{in}d'_{nn}\ell_{jn} + \ell_{in}d_{nn}\ell'_{jn})\right]\bigg/ d_{jj} \tag{15}$$

Unfortunately, it is impossible to calculate the derivative of the D matrix without also calculating the derivative of the matrix L. The result, implied from Eqs. (14) and (15), is that it is more expensive to calculate the derivative of D than it is to calculate D in the first place. Thus it is more efficient to calculate the derivative of D by finite differencing unless high precision is required. This is, indeed, the approach adopted in

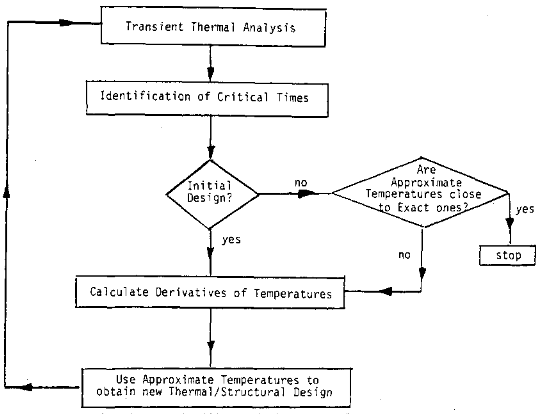

Fig. 1 Flowchart of combined thermal/structural design procedure.

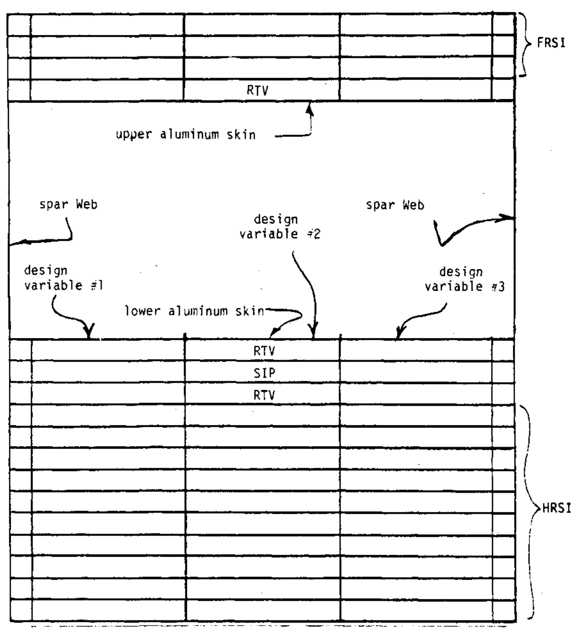

Fig. 2 Thermal finite element model of wing bay.

the present work. In contrast, the calculation of the derivatives of the eigenvalues given by Eq. (6) is fairly inexpensive once the most critical eigenvalues have been calculated. The overall balance of efficiency between the traditional approach and the one suggested here depends strongly on the number of critical eigenvalues, and the number of design variables. However, the proposed approach, based on a finite difference calculation of the derivatives of D, is much more easily implemented.

Optimization Procedure

The structural/thermal design problem is formulated in the general form

$$\text{minimize } m(v)$$

$$\text{such that } \quad g_j(v) \geq 0 \qquad j=1,n \tag{16}$$

where m is the mass of the structure and thermal protection system and $g_j(v)$ is the jth constraint function. The constraints considered here are stress, buckling, and temperature constraints. The design variables are structural and insulation thicknesses. The optimization problem was solved by using an extended interior penalty function formulation employing Newton's method with approximate second derivatives.[16]

To obtain exact values of the constraints a complete thermal/structural analysis is required. A complete thermal/structural analysis consists of the following steps: 1) a transient thermal analysis of the insulated structure; 2) identification of the time when the most critical temperatures occur; and 3) a structural analysis based on these temperatures.

Such a complete analysis may be quite costly. Therefore a design scheme based on two tiers of approximations was used. The general outline of this design approach is shown in Fig. 1. After the first exact thermal analysis is performed the derivatives of the temperatures with respect to the design variables are calculated. These derivatives are used to obtain approximate temperatures during the design process. Following the results of Ref. 15 the approximate thermal analysis was based on the following first-order approximation for a typical temperature component T about a nominal design point v_0

$$T=T(v_0)+\sum_{i\in I_s}\frac{\partial T}{\partial v_i}(v_i-v_{i0})+\sum_{i\in I_T}\frac{\partial T}{\partial v_i}\frac{v_{i0}}{v_i}(v_i-v_{i0}) \tag{17}$$

where I_s is the set of design variables that controls structural thicknesses, I_T the set of design variables that controls insulation thicknesses, and v_i the ith design variable. Equation (17) implies that the temperatures are approximated as a linear function of the structural design variables and the reciprocals of the insulation variables. The approximate temperatures are used to obtain an optimum design using the optimization algorithm of Ref. 16. The final design is then checked by performing a new exact transient thermal analysis and comparing the exact temperatures to the approximate temperatures predicted from Eq. (17). If the difference between the actual and predicted temperatures is small the design procedure is terminated. If, on the other hand, the difference between the exact and approximate temperatures is significant the design cycle is repeated starting with the calculation of derivatives of the temperatures about the new design point. The design procedure actually employs a two-tier approximation process. The approximate temperatures are given by Eq. (17) throughout a design cycle. These temperatures are used to calculate "exact" stress and buckling constraints and their derivatives about 20-40 times in the design cycle (once per one-dimensional search). On one-dimensional searches all constraints are approximated using the conservative constraint approximation of Ref. 17.

Design Study of a Shuttle Orbiter Wing Bay

The design procedure described in the previous section is demonstrated by a design study of the Space Shuttle wing bay (see Fig. 2). The model shown in Fig. 2 represents the Space Shuttle wing in the third bay at wing station 240. The model is a two-dimensional section through the bay and was obtained from the authors of Ref. 18. Only the lower skin was sized in the design study. However, because of the radiation exchange through the cavity it was essential for the thermal analysis to model both parts of the bay. The lower wing skin is covered with high-temperature reusable surface insulation (HRSI) with a strain isolation pad (SIP) lying between the wing skin and the HRSI. The upper skin is covered with felt reusable

Table 1 Material properties for the equivalent orthotropic plate (representing the stiffened aluminum panel) and for the HRSI

	Metal plate	Insulation
E_{11}^{a}	11.9×10^6 psi (82.0 GPa)	0
E_{12}	2.1×10^6 psi (14.5 GPa)	0
E_{22}	6.38×10^6 psi (44.0 GPa)	0
E_{33}	2.13×10^6 psi (14.7 GPa)	0
ρ	0.1 lb/in.^{3} (2770 kg/m^3)	0.0052 lb/in.^{3} (144 kg/m^3)

[a]For bending the effect of the stiffener was approximated by multiplying E_{11} by 50.

Table 2 Design optimization history

Design cycle	Initial design variables: Aluminum, in. (mm)	HRSI, in. (mm)	Weight, lb (mass, kg)	Final design variables: Aluminum, in. (mm)	HRSI, in. (mm)	Weight, lb (mass, kg)	Aluminum temperature: Predicted, °F (K)	Actual, °F (K)
1	0.119 (3.02)	1.30 (33.0)	30.3	0.156 (3.96)	1.16 (29.5)	37.4	350.0	338.0
	0.119 (3.02)		(13.7)	0.166 (4.22)		(17.0)	(450.0)	(443.0)
	0.119 (3.02)			0.188 (4.78)				
2	0.17 (4.32)	1.16 (29.5)	37.4	0.157 (3.99)	1.10 (27.9)	36.6	350.0	349.1
	0.17 (4.32)		(17.0)	0.158 (4.01)		(16.6)	(450.0)	(449.5)
	0.17 (4.32)			0.187 (4.75)				

surface insulation (FRSI). The transient thermal analysis was performed with the SPAR finite element thermal analyzer.[19] The SIP and room temperature vulcanized (RTV) adhesive layers lying on both sides of the SIP were modeled using SPAR K41 elements (4-node, two-dimensional conduction elements). The spar caps, rib caps, and rib trusses were modeled using SPAR K21 elements (2-node, one-dimensional conduction elements) so as to form a frame. The insulation was modeled in ten layers on the lower surface and three layers on the upper surface using SPAR K41 elements. Radiation heat transfer from the upper and lower surfaces and inside the cavity was modeled using R21 elements. The model contained a total of 123 grid points and 191 elements. The heat loads as a function of time are given in Ref. 18.

A 2500-s temperature history was performed using the implicit variable-step Gear algorithm.[20] The critical point was considered to be the point of maximum temperature in the lower skin, which typically occurred around 2000 s. The structural model of the lower skin was modeled with a special purpose finite element program. Prebuckling stresses were calculated using isoparametric 4-node plain stress elements. The bending was represented by a high-accuracy rectangular element.[21] The stiffened aluminum skin was represented by an equivalent orthotropic plate. The material properties for this orthotropic plate are given in Table 1, along with the properties of the HRSI. A six-element model was used to represent the aluminum plate with three elements in the chordwise direction and two elements in the spanwise direction. The prebuckling loads were assumed to be entirely due to restraint against thermal expansion. It was assumed that 20% of the thermal expansion was blocked.

The design variables were the aluminum thickness in three segments of the skin as shown in Fig. 2 and the thickness of the insulation which was assumed to be uniform. The thickness variation between adjacent elements in the aluminum skin was constrained to be less than 0.03 in. (0.76 mm). The allowable aluminum temperature was taken to be 350°F (450 K).

The results of the design study are summarized in Table 2. It is shown that two major design cycles were required. The temperature and their derivatives were first calculated from the model given in Ref. 18 where the insulation thickness was 1.3 in. (33 mm) and the aluminum thickness was 0.119 in. (3.02 mm). The final design obtained based on these derivatives predicted a maximum temperature equal to the allowable maximum of 350°F (450 K). The actual temperatures were about 12°F (7 K) lower. The second design cycle used the derivatives of the temperatures at the final design of the first design cycle. The changes in the design between the first and second design cycles were minor. The difference between the predicted and actual temperature at the final design in the second cycle is very small so that the design procedure could be terminated. These results show that the approximation for the temperatures works very well and thus requires very few design cycles.

The optimum design obtained in the design study showed that the metal worked at its allowable temperature. Thus, a sequential design approach would have produced the same final design, albeit less quickly (because of the need to iterate between the thermal and structural analyst). A similar result was obtained in Ref. 11 for aluminum panels and is due to the large heat capacitance of the aluminum. The aluminum structure works as a heat sink in this case. Increasing the aluminum thickness not only raises the buckling resistance of the skin but also reduces the temperature considerably. For other materials which are considered for use in the Space Shuttle the picture may be different because of their lower heat capacity. Even for aluminum there is no large penalty if the material is required to work at a lower allowable temperature. The design study was repeated with an allowable aluminum temperature of 300°F (422 K). The final weight changed from 36.6 lb (16.6 kg) to 37.6 lb (17.1 kg), which is an increase of less than 3%.

Concluding Remarks

A procedure for simultaneous thermal/structural design has been presented. The procedure has the advantage over sequential thermal/structural design procedures in that it can reduce the effort for obtaining a design. It also can result in designs which are superior to the one that may be obtained by sequential methods. The procedure is based on a first-order approximation for the temperatures which is updated periodically. Constraints are posed on temperatures, stresses, and buckling behavior. The treatment of the buckling constraints is novel in that it does not require the solution of an eigenvalue problem.

The procedure has been demonstrated in a design study of the Space Shuttle Orbiter wing bay. It was found that at the optimum design the aluminum infrastructure works at its maximum allowable temperature, as is usually assumed in sequential thermal/structural design. However, it was also found that the weight penalty for working at lower temperatures is small. Thus, it may be expected that for other materials which have a lower heat capacitance than aluminum such an assumption will not be justified.

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